

INVESTIGATION OF SPECTRAL ANALYSIS TECHNIQUES  
FOR RANDOMLY SAMPLED VELOCIMETRY DATA

by

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## ABSTRACT

It is well known that velocimetry (LV) generates individual realization velocity data that are randomly or unevenly sampled in time. Spectral analysis of such data to obtain the turbulence spectra, and hence turbulence scales information, requires special techniques. The "slotting" technique of Mayo et al [1], also described by Roberts and Ajmani [2], and the "Direct Transform" method of Gaster and Roberts [3] are well known in the LV community. The slotting technique is faster than the direct transform method in computation. There are practical limitations, however, as to how a high frequency an accurate estimate can be made for a given mean sampling rate. These high frequency estimates are important in obtaining the microscale information of turbulence structure. It has been found from previous studies [1-4] that reliable spectral estimates can be made up to about the mean sampling frequency (mean data rate) or less. If the data were evenly sampled, the frequency range would be half the sampling frequency (i.e. up to Nyquist frequency); otherwise, aliasing problem would occur. The mean data rate and the sample size (total number of points) basically limit the frequency range. Also, there are large variabilities or errors associated with the high frequency estimates from randomly sampled signals. Roberts and Ajmani [5] have proposed certain pre-filtering techniques to reduce these variabilities, but at the cost of low frequency estimates. The prefiltering acts as a high-pass filter. Further, Shapiro and Silverman [6] showed theoretically that, for Poisson sampled signals, it is possible to obtain alias-free spectral estimates far beyond the mean sampling frequency. But the question is, how far? During his tenure under 1993 NASA-ASEE Summer Faculty Fellowship Program, the author has investigated from his studies on the spectral analysis techniques for randomly sampled signals that the spectral estimates can be enhanced or improved up to about 4-5 times the mean sampling frequency by using a suitable prefiltering technique. But, this increased bandwidth comes at the cost of the lower frequency estimates. The studies further showed that large data sets of the order of 100,000 points, or more, high data rates, and Poisson sampling are very crucial for obtaining reliable spectral estimates from randomly sampled data, such as LV data. Some of the results of the current study are presented here.

Figures 1-2 show examples of the sine wave investigation. In Fig. 1, a 5000 Hz sine signal was sampled randomly (Poisson) at a mean frequency of only 100 Hz. The results show that the signal is easily detected even though the sampling frequency is 50 times less than the signal frequency. In Fig. 2, the case has been extended to a 10,000 Hz sine wave sampled at only 100 Hz (100 times less). Again, the signal is detected, though with some increased side noise.

Figure 3 shows the normalized spectral (power spectral density, PSD) estimates obtained for a simulated first-order spectrum (FOS) example. The FOS is considered here because it simulates closely a typical one-dimensional turbulence spectrum. PSD estimates were obtained for both unfiltered (circles) and prefiltered (squares) data. It can be seen from the results that, with prefiltering, the spectral estimates have been improved up to about 4-5 times the mean sampling frequency. The original unfiltered data provides reasonable estimates only up to about the mean sampling frequency or less. The high-pass filtering effect on the prefiltered data can be clearly seen from the results.

Figure 4 shows the normalized PSD estimates from Poisson and non-Poisson samples of a simulated FOS. It can be seen from the results that the estimates from non-Poisson samples deviate considerably from the true values, but do give useful information below about half the mean sampling frequency. The study further showed that the data from non-Poisson sampling were not amenable to spectral improvements by prefiltering techniques.

Further research is required to determine the effects of particle dynamics, nonhomogeneous seeding, velocity bias, etc. on the spectral estimates and to establish the accuracies and the confidence limits of these techniques. Research is also needed to extend the spectral analysis techniques to obtain cross-correlation and cross-power spectral estimates from randomly sampled 2-D/3-D LV experimental data.

#### REFERENCES

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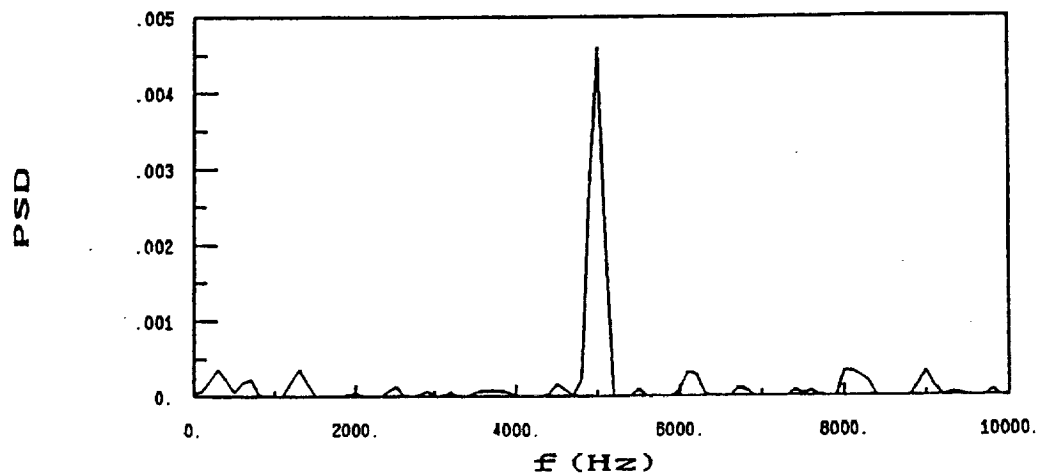


Fig. 1 Example of a 5000 Hz sine wave sampled (Poisson) at a mean sampling rate of 100 Hz. The signal is detected by the slotting technique of spectral analysis even though the sampling rate is 50 times less than the signal frequency. (PSD = Power Spectral Density)

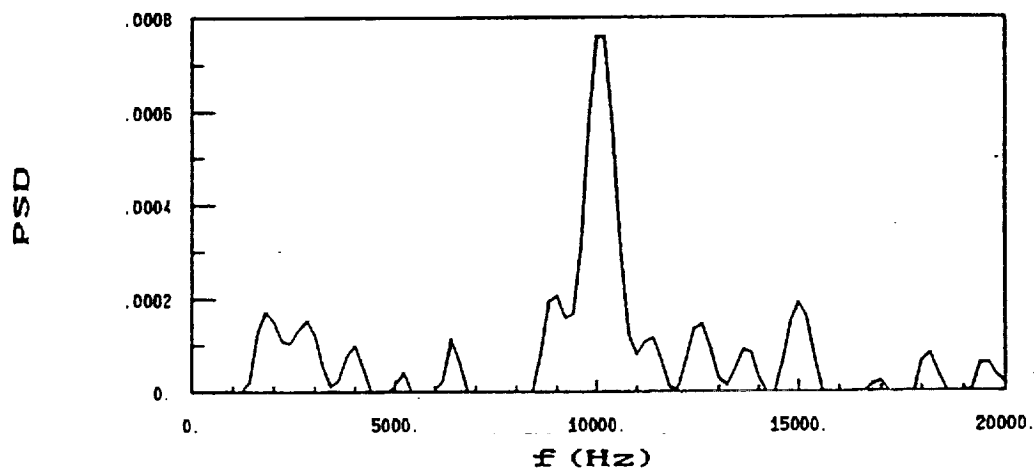


Fig. 2 Example of a 10,000 Hz sine wave sampled (Poisson) at a mean sampling rate of only 100 Hz (100 times less). The signal is still detected fairly by the slotting technique.

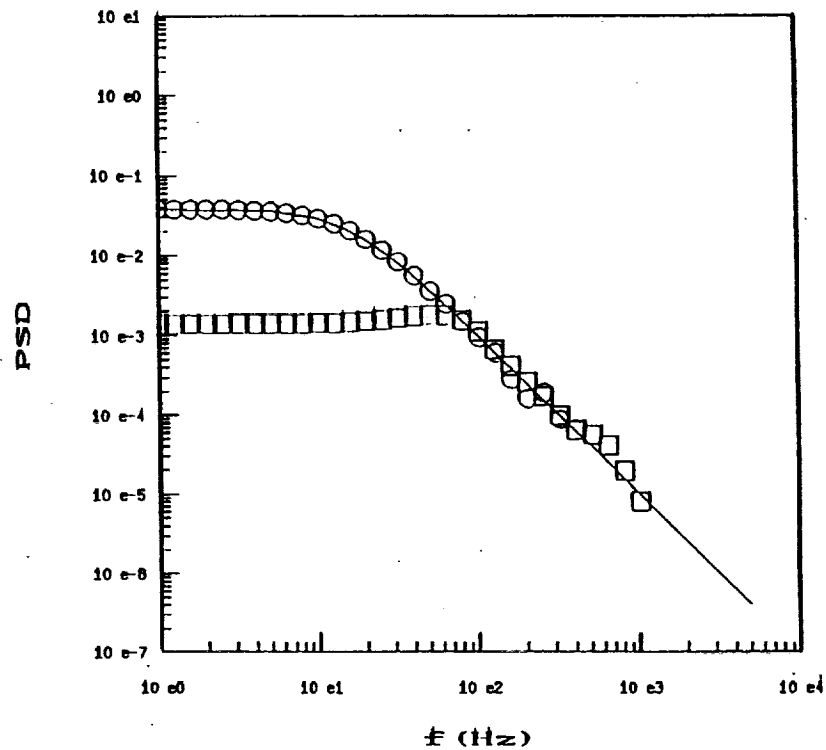


Fig. 3 Example of a simulated first-order spectrum (FOS) sampled at a mean sampling rate of 200 Hz. Solid line represents the theoretical spectrum. Circles denote the spectral estimates of the original unfiltered data and squares that of the prefiltered data. Note the high-pass filtering effect and the improvement of the spectral estimates up to about 4-5 times the sampling frequency.

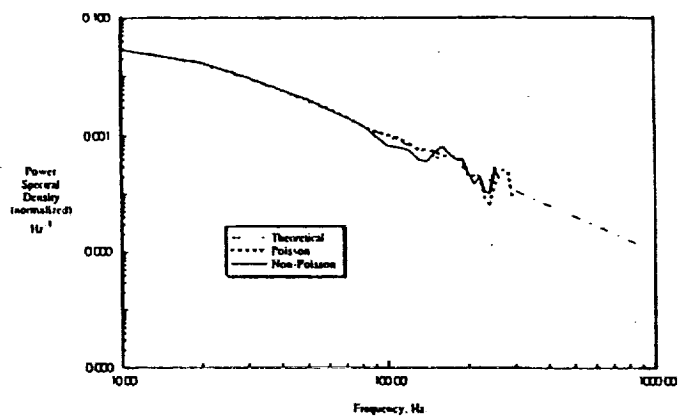


Figure 4. Power Spectra of Poisson and Non-Poisson Data of  $\lambda$  Simulated First-Order Spectrum. Data Rate = 200/s.